B. Sc. Part II (Hons.)

4th paper (Diff. Eqns).

LDECC

Q. solve $(D^2-1)y = \cosh x \cos x$

Soln For CF $D^2-1=0 \Rightarrow D = \pm 1$

$\therefore CF = c_1 e^x + c_2 e^{-x}$ where c_1 and c_2 are two arbitrary constants.

For PI

$$P.I. = \frac{1}{D^2-1} \cosh x \cos x$$

$$\because \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\Rightarrow PI = \frac{1}{2(D^2-1)} (e^x + e^{-x}) \cos x$$

$$= \frac{1}{2(D^2-1)} (e^x \cos x + e^{-x} \cos x)$$

$$\Rightarrow PI = \frac{1}{2(D^2-1)} e^x \cos x + \frac{1}{2(D^2-1)} e^{-x} \cos x \quad \text{---(1)}$$

Now, $$\frac{1}{2(D^2-1)} e^x \cos x = e^x \frac{1}{2[(D+1)^2-1]} \cos x$$

$$= \frac{e^x}{2} \cdot \frac{1}{(D^2+2D)} \cos x$$

$$\therefore \frac{1}{2(D^2-1)} e^{x} \cos x$$

$$= \frac{e^x}{2} \frac{1}{D^2+2D} \cos x$$

$$= \frac{e^x}{2} \cdot \frac{1}{-1^2+2D} \cos x = \frac{e^x}{2} \frac{1}{(2D-1)} \cos x$$

$$= \frac{e^x (2D+1)}{2(2D-1)(2D+1)} \cos x$$

$$= \frac{e^x}{2} \cdot \frac{(2D+1)}{(4D^2-1)} \cos x$$

$$= \frac{e^x}{2} \left[\frac{2D+1}{4\times(-1^2)-1}\right] \cos x = \frac{e^x}{-10} (2D+1) \cos x$$

$$= \frac{e^x}{-10} [2D(\cos x) + \cos x] = \frac{e^x . (\cos x - 2\sin x)}{-10}$$

Also $\frac{1}{2(D^2-1)} e^{-x} \cos x = e^{-x} \frac{1}{2[(D-1)^2-1]} \cos x = \frac{e^{-x}}{2(D^2-2D)} \cos x$

$$= \frac{e^{-x}}{2(-1-2D)} \cos x = \frac{e^{-x}}{-2(2D+1)} \cos x$$

$$= \frac{e^{-x}}{-2(2D+1)} \cdot \frac{2D-1}{2D-1} \cos x = \frac{e^{-x}}{-2(4D^2-1)} (2D-1) \cos x$$

$$= \frac{e^{-x}}{-2(-4.1^2-1)} [2D(\cos x) - \cos x] = \frac{e^{-x}}{10} (-2\sin x - \cos x)$$

$$= \frac{e^{-x}}{-10} (\cos x + 2\sin x).$$

$$\therefore PI = \frac{e^x}{-10} (\cos x - 2\sin x) + \frac{e^{-x}}{10} (\cos x + 2\sin x).$$

Hence soln = CF + PI